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Indirect Determination of Heat Flux to a Lander Heat Protection System

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A feasibility of the heat transfer history determination using the end-point state of irreversible processes in a spacecraft heat protection system is considered. The special inverse conduction problem in materials of heat shield is discussed. The requirements are formulated for the composition and accuracy of measurements within the heat protection material. The parameterization of the heat flux is performed by means of specified functions for laminar and turbulent flow, that allow one to reduce the number of parameters and improves the accuracy of the approximation. The results of numerical experiments corroborate a feasibility of the heat flux reconstruction if additional a priori information regarding trajectory parameters is present.

Nomenclature

C = specific heat
$H(t)$ = flight altitude, km
h = thermal effect of destruction reaction
I_0 = enthalpy of gas at boundary-layer edge
$K_1 \div K_4$ = parameters of amplitude type
k = gas-permeability coefficient, Darcy
M_f = heat protection material mass in the endpoint state (per surface area unit) in zone of measurement
P = pressure
Q = heat flux, kW/m^2
$Q_{st}(t)$ = heat flux at the stagnation point of sphere, kW/m^2
$Q_{t\,max}(t)$ = maximum turbulent heat flux at sphere, kW/m^2
r = volume portion
T = temperature, K
t = time, s
t_{stab} = conventional time of capsule stabilization, s
t_{tr} = time of transition to turbulent flow regime, s
V = velocity
x = coordinate
α = heat transfer coefficient
ε = emissivity
$\varepsilon(K_1)$ = discrepancy of computed and measured data
λ = thermal conductivity
μ = viscosity
Π = porosity
ρ = density, kg/m^3
σ = Stefan-Boltzmann's constant

Subscripts

f = final state parameters
g = gas
i = number of destruction process reaction (stage)
l = laminar
m = component of material
max = maximum
s = solid matrix of material
t = turbulent
W = wall condition
Σ = total values for composite material
0 = initial condition

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Introduction

HEAT flux measurements on the surface of a re-entry vehicle have importance from the viewpoint of payload weight and of spacecraft reliability. However, the measurements are limited by telemetry channels shortage and cost. Inserting sensors into a heat protection material can break its integrity; measurements by means of thermocouples in the material during thermal decomposition contain relatively large errors. By means of numerical methods one can easily obtain valid results for the heat flux in certain zones (e.g., at the sphere stagnation point). In general, for the heat flux determination one should solve the three-dimensional problem for the viscous and reactive gas flow with heat protection decomposition and injection into the boundary layer. Unfortunately, a number of parameters are not known precisely. They are, e.g., a time of turbulent transition or the magnitude of surface roughness during the decomposition.

In this connection, the different types of thermoindicators and methods of the heat protection coating structure analyses are implemented for the heat flux history reconstruction. Usually, these data provide some indirect information about the heat flux (e.g., a maximum temperature of some structure members). A feasibility of the heat transfer history reconstruction [$Q_W(t)$ or $T_W(t)$], using irreversible changes in thermosensitive materials is considered in Refs. 1 and 2. As the input data, the maximal temperature profile (provided by means of thermoindicators) was used in Ref. 1, and the density profile of a heat protection material was used in Ref. 2.

In the present study, the feasibility of heat flux $Q_W(t)$ reconstruction with utilization of thermosensitive materials[1,2] is considered regarding the heat protection coating of a lander. The re-entry ballistic capsule (a spherically blunt-nosed biconic, Fig. 1) can be considered as a generic example. Typical velocity, altitude, and heat flux are presented in Fig. 2. The capsule heat protection system contains an external layer made of glass-fiber plastic with a phenol-formaldehyde binder and quartz-kapron textile base. The quartz felt internal layer is attached to the aluminum shell.

It is assumed that before a flight experiment one should perform the following preparation:

1) Install thermoindicators on the internal surface of shell
2) Locate thermoindicators in the heat shield insulation normal to the heated surface.[1]

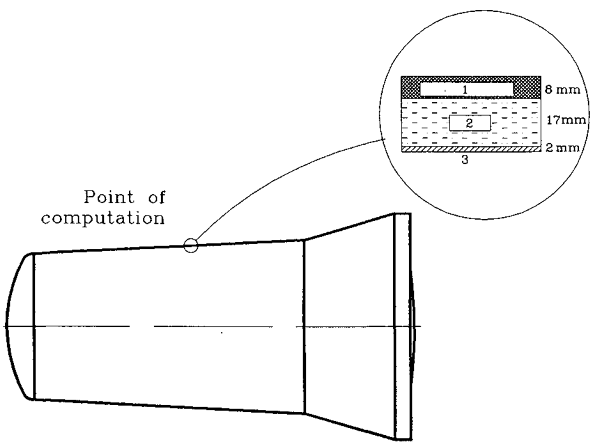

Fig. 1 Sketch of ballistic capsule and its heat protection: 1, glass fiber plastic, 2, heat insulation, and 3, shell.

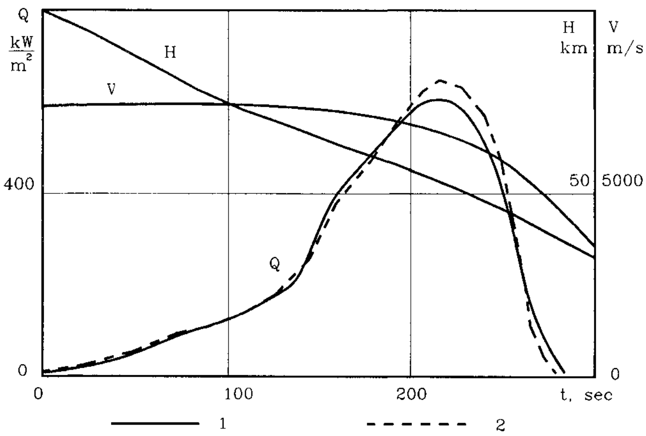

Fig. 2 Heat flux and typical trajectory parameters: 1, heat flux value and 2, result of heat flux approximation by six parameters.

After the flight one should perform following operations:

1) Read the thermoindicators information and determine: T_f – aluminum shell maximum temperature and $T_{max}(x)$, maximum temperature in the depth of the insulation layer.

2) Weigh the heat protection sample and determine M_f, heat protection material mass per surface area unit.

3) Determine the density profile $\rho(x)$ in the external layer (e.g., by filing down the sample).

The mathematical model of the thermal history reconstruction contains an algorithm for the calculation of the temperature distribution and heat protection material properties variation and an algorithm for optimization. It allows one to minimize the discrepancy between calculated data and experimental ones by changing coefficients parameterizing the heat flux. The reconstruction of the heat flux in piecewise linear functions is presented in Refs. 1 and 2. In this study the parameterization of the heat flux is performed by means of specified functions for laminar and turbulent flow, which allows one to reduce the number of parameters and improves the accuracy of the approximation.

Parameterization of the Heat Flux

It is stated in Refs. 1 and 2 that irreversible changes in the heat protection material structure retain information about some number of parameters. An important issue (from the viewpoint of calculation time) is the parameterization of the unknown heat flux by the minimum number of parameters with the best precision. The accuracy of the heat flux approximation depends on a priori information presence:

1) Trajectory parameters [altitude $H(t)$ and velocity $V(t)$] allow one to evaluate gas parameters near the capsule's surface and heat flux values at reference points. As the specified functions for laminar regime the $Q_{st}(t)$ and $Q_{t\max}(t)$ are accepted.

2) Turbulent transition occurs at some unknown time t_{tr}.

3) The laminar heat flux can't be greater than the turbulent one (after transition heat flux increases) for equal flow parameters.

4) Angle-of-attack oscillations increase the average heat flux value and decay at time t_{stab}.

The minimal parameter number for problems of this type is 3: a time of turbulent transition and two amplitudes (for laminar and turbulent regimes). In this form the problem is valid for a re-entry spacecraft of the "Soyuz" type. For these spacecraft types, appreciable angle-of-attack oscillations are absent.

The problem considered in this study is more general since angle-of-attack oscillations are included. At re-entry, the capsule performs attenuating oscillations that increase the average heat flux. Therefore, the approximation has been carried out by means of six parameters as follows:

The turbulent transition was approximated with a step function:

$$Q_W(t) = K_1 \cdot Q_l(t) + (1 - K_1) \cdot Q_t(t)$$
$$K_1 = 1 \quad \text{if} \quad t < t_{cr} \qquad \text{and} \qquad K_1 = 0 \quad \text{if} \quad t > t_{tr} \tag{1}$$

Angle of attack oscillation influence in the laminar regime:

$$Q_l(t) = [K_1 + (K_2 - K_1) \cdot t/t_{stab}] \cdot Q_{st}(t) \qquad t < t_{stab} \tag{2}$$

$$Q_l(t) = K_2 \cdot Q_{st}(t) \qquad t > t_{stab} \tag{3}$$

and in the turbulent regime:

$$Q_t(t) = [K_3 + (K_4 - K_3) \cdot (t - t_{tr})/(t_{stab} - t_{tr})] \cdot Q_{t\max}(t)$$
$$t < t_{stab} \tag{4}$$

$$Q_t(t) = K_4 \cdot Q_{t\max}(t) \quad \text{if} \quad t > t_{stab} \tag{5}$$

Values K_1, K_2, K_3, and K_4 (heat flux amplitudes), t_{tr} (time of turbulent transition), t_{stab} (time of oscillations termination) form six-component vector $\boldsymbol{K}_i$, parameterizing the heat flux. Results in Fig. 2 show the relatively good quality of the approximation.

Mathematical Model of Heat Transfer

The heat protection coating contains two layers, an external heat protection material and internal heat insulation one. In the external layer, components are available that decompose during heating with the production of gases. The decomposition process is described by eight parallel-independent reactions. Thermophysical properties of the material are calculated using the known properties of the components with the account of temperature and the extent of decomposition dependence.

The system of equations describing energy and mass transfer is based on the following assumptions:

1) The transfer of energy and mass within the material is due to convection, heat conduction, and radiation.

2) External material ablation (thickness change) is absent.

3) Temperature and density gradients normal to the surface are greater than ones along the surface.

4) The gas transfer is described by Darcy's equation.

5) The gas temperature in the porous material is in equilibrium with the temperature of the matrix

$$\frac{\partial}{\partial t}\left(\frac{\rho_g v_g^2}{2} + C_\Sigma \rho_\Sigma T\right) = \sum_{m,i} h_{mi} \frac{d\rho_{smi}}{dt}$$
$$- \frac{\partial}{\partial x}\left(P_g v_g + \rho_g v_g C_{pg} T + \frac{\rho_g v_g^3}{2} - \lambda_\Sigma \frac{\partial T}{\partial x}\right) \tag{6}$$

$$\frac{\partial \rho_g}{\partial t} = \frac{\mathrm{d}\rho_g}{\mathrm{d}t} - \frac{\partial}{\partial x}(\rho_g v_g) \tag{7}$$

$$\frac{\mathrm{d}\rho_g}{\mathrm{d}t} + \frac{\mathrm{d}\rho_s}{\mathrm{d}t} = 0, \qquad \frac{\partial \rho_s}{\partial t} = \sum_{m,i} \frac{\mathrm{d}\rho_{\mathrm{smi}}}{\mathrm{d}t}$$

$$i = 1, \ldots, 8, \qquad m = 1, 3 \tag{8}$$

$$\frac{\mathrm{d}\rho_{\mathrm{smi}}}{\mathrm{d}t} = -\rho_{\mathrm{smi}}^{n_{\mathrm{mi}}} A_{\mathrm{mi}} \exp\left(-\frac{E_{\mathrm{mi}}}{RT}\right)$$

$$i = 1, \ldots, 8, \qquad m = 1, 3 \tag{9}$$

$$\rho_g = P_g \frac{M_g \Pi}{RT}, \qquad v_g = -\frac{k}{\mu}\frac{\partial P_g}{\partial x} \tag{10}$$

$$k = k_0 \left(\frac{\Pi}{\Pi_0}\right)^3 \left(\frac{1-\Pi_0}{1-\Pi}\right)^2, \qquad \Pi = 1 - \left(\sum_{m,i} \rho_{\mathrm{smi}}\right) \Big/ \rho_{s0}$$

$$\lambda_\Sigma = \lambda_\Sigma(r_m, \lambda_g, \lambda_{\mathrm{sm}}), \qquad C_\Sigma \rho_\Sigma = C_s \rho_s (1 - \Pi) + C_{\mathrm{pg}} \rho_g \Pi$$

$$C_s \rho_s = \sum_{m,i} C_{\mathrm{sm}} \rho_{\mathrm{smi}}, \qquad \lambda_g = f_1(T, \Pi)$$

$$\lambda_{\mathrm{sm}} = f_2(T), \qquad C_{\mathrm{sm}} = f_3(T), \qquad C_{\mathrm{pg}} = f_4(T)$$

$f_1(T, \Pi)$, $f_2(T)$, $f_3(T)$, and $f_4(T)$ are known functions for substances that compose the material. The λ_Σ is computed in accordance with generalized conductivity method.[3]

The following boundary condition is posed on the external surface:

$$\left.\frac{\partial T}{\partial x}\right|_{x=x_W} = -\alpha(t)/C_p \cdot [I_0(t) - I_W(t)] - \varepsilon \cdot \sigma \cdot T_W^4 \tag{11}$$

The internal surface of heat protection system is insulated:

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0 \tag{12}$$

For the mass transfer equation the boundary condition on the external surface is a pressure $P_W(t)$, on the interface with insulation the nonleakage condition ($\partial P/\partial x = 0$) is posed.

This model is realized with the finite difference method (second accuracy order in space, first in time) in the Fortran.

Heat Flux Reconstruction Problem

At heating, irreversible changes occur in the capsule heat protection material and on its structural members. These changes are a thermodestruction of the heat protection material and a fusion of thermoindicators. In the endpoint state these changes retain information about the heat flux acting on the capsule's surface in flight. In order to use this information we should calculate the temperature field and density value in the heat protection material. The heat flux is parameterized by some values K_i: $Q_W(t) = F(K_i)$. For a given heat flux, we solve the direct heat conduction problem [Eqs. (1–12)] and calculate the endpoint state: $[T_f, T_{\max}(x), \rho(x), M_f]$. Thereafter we compare these values with results of measurements on the real capsule. Values with marker $\sim(\tilde{\rho})$ are experimental data. As a result, we find the discrepancy of calculated and measured data:

$$\varepsilon(K_i) = \left(\frac{T_f - \tilde{T}_f}{T_f}\right)^2 + \left(\frac{M_f - \tilde{M}_f}{M_f}\right)^2 + \int \left[\frac{\rho(x) - \tilde{\rho}(x)}{\rho(x)}\right]^2 \cdot \mathrm{d}x + \int \left[\frac{T_{\max}(x) - \tilde{T}_{\max}(x)}{T_{\max}(x)}\right]^2 \cdot \mathrm{d}x \tag{13}$$

If, at some set of K_i, the laminar heat flux is greater than the turbulent one $[Q_1(t) > Q_t(t)]$, then this result is considered a nonphysical, and the "penalty" function (which is proportional to $|Q_1(t) - Q_t(t)|$), is introduced into the discrepancy.

We determine parameters K_i by means of optimization problem solution:

$$K_i = \arg\min[\varepsilon(K_i)] \tag{14}$$

The precision of problem [Eqs. (1–14)] solution was studied by an initial data perturbation with a random error.

Results of Calculation

The spatial grid of 19 nodes and about 1000 time steps are used for the direct heat conduction problem solution. The discrepancy gradient $\partial\varepsilon/\partial K_i$ is calculated by a finite difference method. The optimization is performed by means of a steepest descent method. Ten iterations of the steepest descent method consume about one 1 h of PC/386 computer time.

The solution of the one-parameter problem for determining the t_{tr} is presented in Fig. 3 in the form of the discrepancy ε dependence on time t_{tr}. Similar dependencies are found for other approximation parameters. For heat flux $Q_W(t)$ value the calculation result is presented in Fig. 4. The gradient method allows one to find this solution with 3–5 iterations and with good precision (0.1%).

For the six-parameter approximation, the gradient method does not provide such good results. Within a few tens of iterations, the minimization process yields a solution with a maximum error of about 5%. The reason for this is the discrepancy shape. The discrepancy isolines for t_{tr} and laminar

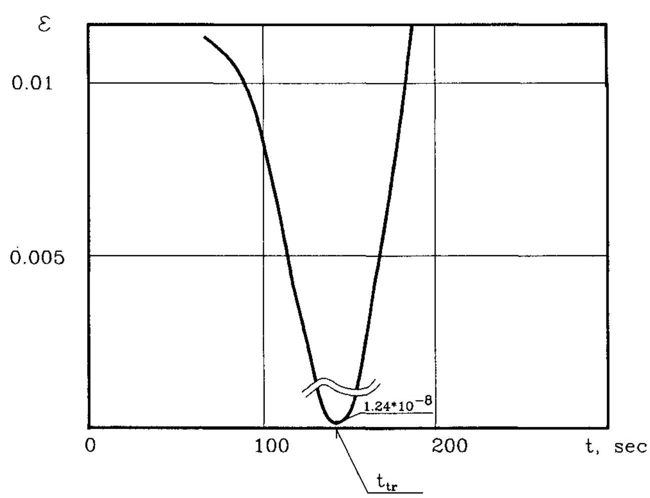

Fig. 3 Discrepancy profile for t_{tr} determination.

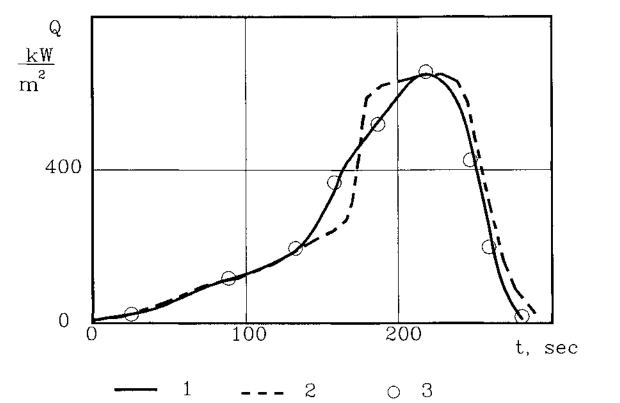

Fig. 4 One-parameter heat flux reconstruction results: 1, result of calculation, 2, initial guess, and 3, exact solution.

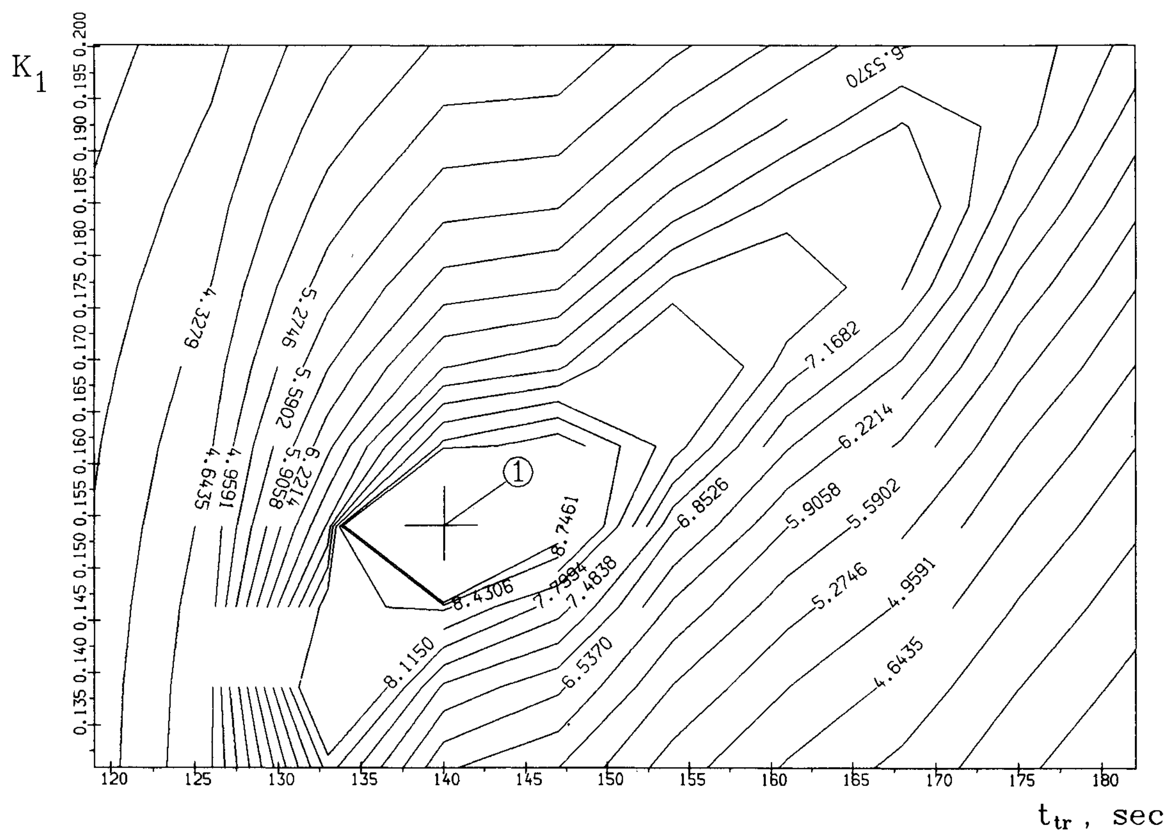

Fig. 5 Discrepancy shape ε in plane K_1, t_{tr} for exact solution: 1, exact solution.

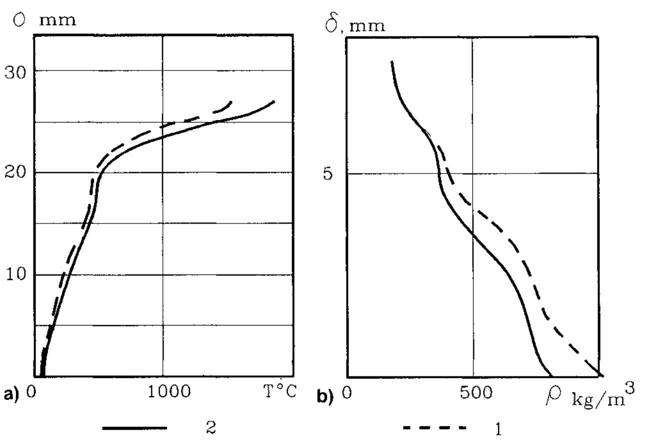

Fig. 6 Initial data used in the problem: a) maximal temperature profile in depth; 1, precise solution and 2, initial guess and b) density profile in depth; 1, exact solution and 2, initial guess.

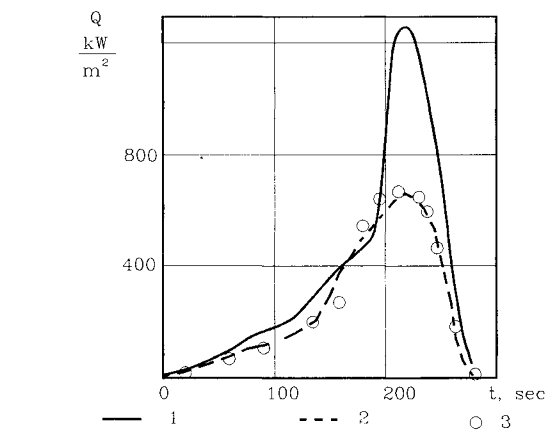

Fig. 7 Results for six-parameter heat flux reconstruction: 1, initial guess, 2, exact solution, and 3, results of calculation.

heat flux amplitude K_1 for precise initial data are presented in Fig. 5. The discrepancy profile has a deep "ravine." If we vary two parameters, then the effect of a mutual compensation arises. The direction exists that locally coincides with one, tangent to the surface of constant total heat flux. The parameter variation along this direction changes the discrepancy less than the variation normal to this direction (see Fig. 5). For "precise" (four digits after the decimal point) initial data [see profiles of $T(x)$ and $\rho(x)$ in Fig. 6] the heat flux was reconstructed. The results are presented in Fig. 7 in terms of the exact heat flux, the calculated results, and the initial guess.

To estimate the results accuracy the reconstruction of the heat flux was made for disturbed (1% random error dispersion) initial data. For a one-parameter T_{tr} approximation the solution error is about 3%. For the six-parameter case the results contain a maximum error of about 5%. Initial data error of 3% provides the results error of about 10%. For the data error of 10% the picture of the discrepancy profile, presented in Fig. 5 changes qualitatively, and the solution (minimum) disappears.

The results of calculations show that the discussed problem can be solved successfully if we have precise initial data (with error dispersion about 1%). The high sensitivity of the problem is defined by two circumstances. The first one is the attenuating property of heat conduction equation that intensively damp high-frequency harmonics. The second one is the closeness of $Q_l(t)$ and $Q_t(t)$ values at transition time t_{tr}, and the uncertainty of selection connected with this. The latter is the turbulization [the greater the difference between $Q_l(t)$ and $Q_t(t)$ at the transition time of t_{tr}], the higher is the precision of t_{tr} evaluation.

If there are not enough precise experimental data for correct evaluation of t_{tr}, then we could describe the heat flux by

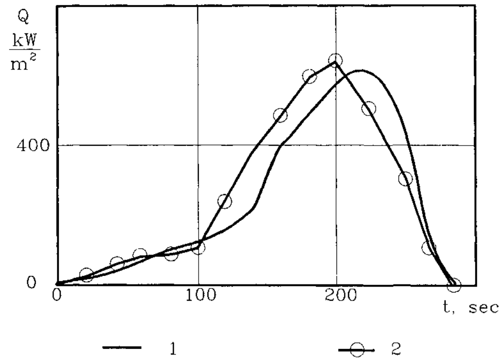

Fig. 8 Results of heat flux reconstruction (piecewise linear approximation by five parameters): 1, exact solution and 2, results of calculation.

the piecewise linear approximation, that is more stable to error. The results of the heat flux evaluation at an error dispersion of 10% are presented in Fig. 8. The heat flux is approximated piecewise linearly by 5 parameters and determined with the error about 30%.

Discussion

We have a variational statement of some "boundary-retrospective" inverse problem of heat transfer. Similar to all inverse problems, this problem is an ill-posed and unsteady one. The heat flux, retained by the heat protection system, passes through double smoothing by use of the equation of heat conduction and time integration, therefore, small changes of internal parameters could correspond to large changes of heat flux.

The problem of uniqueness for this problem at enough arbitrary $Q_W(t)$ is not solved. In Ref. 1 some functions classes are presented, for which the problem of heating history reconstruction by means of maximal temperature profile is nonunique. In this article, the detection of minima, which can be connected with nonuniqueness of a solution, was carried out by numerical experiments. The results were displayed by isolines of the discrepancy ε in the plane of control parameters. The indications of ambiguity at the initial data error of about 1–3% are absent.

In the real flight, a number of errors (e.g., the error of heat flux approximation and the error of the heat transfer model in the heat protection system), can be added to a measurement error. Consequently, the total precision of the method can be determined only by means of flight experiments.

Concluding Remarks

Results of numerical experiments demonstrate the feasibility of heating history reconstruction if we know the endpoint state of a heat shield structure with thermoindicators located in depth and with the thermodestruction of the external layer. The set of values measured in the endpoint state of such heat protection should include the depth dependence of maximal temperature and density profiles with the precision about 1%.

If we have initial data with a relatively high precision (1%) then we can evaluate the t_{tr} and the amplitudes of laminar and turbulent heat flux. If we have initial data with low precision (10% error) then we can evaluate the piecewise linearly approximated heat flux $Q_W(t)$. In any case, this method can be useful for the determination of abnormal situations when the heat flux can greatly differ from a nominal value.

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